Sammenligning av geoidens høyde utledet fra geopotensialmodellen EGM2008 med GPS/nivellementsdata i et testområde i Iran

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Hossein Nahavandchi and Hossein Mahdavipour: Comparison of geoid heights from the EGM2008 geopotential model and GPS / levelling data in a study area in Iran

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The global Earth Gravity Model EGM2008 is evaluated locally using Global Positioning System (GPS)/levelling data in a test area in Iran. The non-tidal release of the EGM2008 is used in the computations. An indirect approach of geoidal height determination is investigated. The indirect approach is based on computation of a height anomaly ζ from EGM2008 geopotential coefficients, and the subsequent conversions to a geoid undulation N. Appropriate corrections are applied to height anomalies to convert them to geoidal heights. The indirect approach reduces the concern of evaluation of geopotential inside the topographic masses. The computation point in this approach is at a point on or above the Earth's surface. The results of computations show that this indirect approach yields good agreement with the GPS/levelling derived geoid heights. As expected, the application of correction terms improves the agreement of geoidal heights at GPS/levelling stations. Comparison of geoidal heights at 33 GPS/levelling stations in a test area in Iran shows an average difference of 40 cm with Standard Deviation (SD) of ± 35 cm.

Key Words: Geoid, Earth Gravity Model, Stokes' formula, GPS/levelling, Topographic corrections

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Introduction

The geoid is the equipotential surface of the Earth's gravity field more or less coinciding with mean sea level and is used as the vertical datum for orthometric heights in many countries. Civil engineers use it as the reference surface for elevations, while oceanographers use it for studies of ocean circulation, currents and tides. It is also valuable to geophysicists for geodynamics studies, geophysical interpretation of the Earth's crust, and prospecting. These types of applications require knowledge of the geoid with a precision of ±1–10 cm. An accurate solution for the geoid determination in physical geodesy has usually been found using Stokes' well-known formula (Stokes, 1849; Heiskanen and Moritz, 1967, p.15) for the anomalous gravity potential, with the geoidal height calculated through Bruns's formula. When solving the Stokes problem, gravity anomalies over the entire Earth are strictly speaking required. However, in practice, the data availability is limited to some spatial domain around the computation point. To obtain such high accuracy for the regional computation of the geoid, a modification of Stokes' formula combines local terrestrial gravity observations and the EGM-derived long-wavelength component of the geoid. However, with the increasing accuracy of the geopotential coefficients derived from the global EGM and the maximum degree of expansion to higher order, the computation of geoidal heights from global Earth gravity models alone has been an issue of increasing importance in the geodetic community.

Terrestrial gravity data in some developing countries have been observed over a long time span, using different equipment, methods and reference frames. As a consequence, they are likely to be affected by various systematic errors. It is obvious that it is more or less impossible to reach cm geoid accuracy with these types of terrestrial gravity data. The main purpose of this paper is to investigate the accuracy of geoidal height computations based on global gravity models alone. In this approach we use the best available model, EGM2008 (Pavlis et al., 2008). Rapp (1971, 1994a, b) has examined different approaches to geoidal height computation using geopotential coefficients of global Earth gravity models. Rapp (1997) and Nahavandchi (2002) noted use of the difference between height anomaly (ζ) and geoidal height (N) and a height anomaly gradient correction term. Smith (1998) and Smith and Small (1999) have investigated use of geoidal height determinations using EGM96 (Lemoine et al., 1997) geopotential coefficients alone together with correction terms.

We demonstrate the efficiency of the approach used in this study using a set of GPS/levelling stations in a test area in Iran. We make geoid height comparison with GPS/levelling data to show that global gravity models alone can also provide reasonable accuracy for geoid heights. Iran has limited ground gravity data and they are sparse and of diverse quality. One of the latest gravity databases gathered in Iran includes 26125 points and mean gravity values (see Kiamehr, 2006). Considering the total area of Iran, 1 648 195 km^2, we have about one gravity point per 63 km^2 on average. It seems that most of the areas in the central desert and mountainous areas will not be explored in the near future.

Rapp (1997) compared the geoidal heights at 960 GPS/levelling stations over the USA. The OSU91A potential model (Rapp et al., 1991) was used. The Root Mean Square (RMS) of differences was ±56 cm. Smith and Small (1999) found an RMS of differences of ±62 cm at 31 GPS/levelling stations in the Caribbean. The EGM96 geopotential model was used and high-frequency corrections to the EGM96 model were also applied. Nahavandchi (2002) used 56 GPS/levelling stations in two test areas in Iran to compare the GPS/levelling geoid with the geoid determined through height anomalies using EGM96 geopotential coefficients. Two correction terms were also applied. The SD and RMS of differences were computed to be ±40 cm and ±61 cm.

Several recent studies have evaluated EGM2008. Gruber (2009) reported an RMS of differences between EGM2008 and GPS/levelling geoids varying between ±3.8 cm (for 675 GPS/levelling stations in Germany) and ±33.4 cm (for 5168 GPS/levelling stations in the USA). Jekeli et al. (2009) found an RMS of differences of ±20.1 cm between the EGM2008 geoid and the geoid derived from GPS/levelling data over 500 irregularly distributed stations in South Korea. Blitzkow and de Matos (2009) reported an RMS of differences of ±68 cm at 1190 GPS/levelling stations throughout South America. In another study, an RMS of differences of ±49 cm was found at 189 GPS/levelling stations in the Baltic countries between the EGM2008 and the GPS/levelling geoid heights (Ellmann et al., 2009).

2. Geoid determination through height anomalies

The approach used in this study is based on computation of a height anomaly ζ, and subsequent conversion to a geoid undulation N. The height anomaly ζ at the surface, i.e. not at the geoid, is computed from geopotential coefficients (the indirect geoid computation). This indirect procedure reduces the concern of evaluation of geopotential on the geoid which on the continents is generally within the topographic masses. In determining the geoid, directly and not through height anomalies, an exterior-type boundary value problem is generally solved. It deals with the external gravity field of the Earth, implying that the gravitational potential is a harmonic function. As the geoid is roughly related to the undistributed sea level and its continuation inside the continents, the harmonic or analytical downward continuation of the external harmonic observables of gravity to the geoid will be biased in continental areas. This means that in applying the external type representation of geopotential coefficients to the continental geoid, a topographic bias due to non-harmonicity in the formulas is expected. This bias can be estimated by removing topographic effects (such that we can then continue downwards the harmonic series of the potential to the geoid), and then restoring the masses. An absolute global mean value of 11 cm and a maximum value of 3.4 m at the Himalayas were calculated for the total removal and restoration of topographic masses. See Nahavandchi and Sjöberg (1998). They used Helmert's second condensation method to calculate theses values, which means that the masses outside the geoid are condensed as a surface layer at sea level in a spherical approximation of the geoid. In the present study, the geoid is computed through height anomalies at the surface, and then the geoid computed is not affected by this topographic bias. The approach is realized according to Heiskanen and Moritz (1967, p. 327); Sjöberg (1995); Rapp (1997) and Nahavandchi (2002):

$$N(\phi,\lambda) = \zeta_E(r,\phi,\lambda) + \frac{\partial\zeta}{\partial r}h + \frac{\partial\zeta}{\partial\gamma}\frac{\partial\gamma}{\partial h}h + \frac{\Delta g_{\text{Bouguer}}}{\bar{\gamma}}H \tag{1}$$

Explanation of this equation follows Eqs. (1a) and (6). See also Fig. 1. As described in Heiskanen and Moritz (1967, p. 327), the conversion of a height anomaly ζ to a geoid height N is done through the following formula at a point P on or above the surface of the Earth:

$$N(\phi,\lambda) = \zeta(r_P,\phi,\lambda) + \frac{\Delta g_{\text{Bouguer}}}{\bar{\gamma}}H \tag{1a}$$

Note that in Eq. (1a) ζ depends on r_P (the geocentric distance to the point *P)*. To improve computation efficiency, we calculate ζ_E (an ζ value) by letting r be the geocentric radius of the point on the surface of the ellipsoid (ellipsoid radius). The second and third terms on the right-hand side of Eq. (1) are necessary correction terms to get the value of ζ at point P from ζ_E. The height anomaly ζ_E can be computed from geopotential coefficients by letting r be the ellipsoid radius and using equation $\zeta = \frac{T}{\gamma}$ (Heiskanen and Moritz, 1967, p. 108)

$$\zeta_E(r,\phi,\lambda) = \frac{GM}{r\gamma}\sum_{n=0}^{n_{\max}}\left(\frac{a}{r}\right)^n \sum_{m=0}^{n}\left(\bar{C}_{nm}\cos m\lambda + \bar{S}_{nm}\sin m\lambda\right)\bar{P}_{nm}(\sin\phi) \tag{2}$$

where

$$\bar{C}_{nm} = \begin{cases} \bar{J}_{nm} - \bar{J}_n^{(N)} & \text{for m=0} \\ \bar{J}_{nm} & \text{for m} \neq 0 \end{cases} \tag{3}$$

and

$$\bar{S}_{nm} = \bar{K}_{nm} \tag{4}$$

Also,

$$\bar{J}_n^{(N)} = \begin{cases} 0 & \text{for odd n} \\ (-1)^{\frac{n}{2}} \dfrac{3e^n\left(1 - \dfrac{n}{2} + \dfrac{5}{2}\dfrac{J_2^{(N)}}{e^2}n\right)}{(n+1)(n+3)\sqrt{2n+1}} & \text{for even n} \end{cases} \tag{5}$$

where $\bar{J}_n^{(N)}$ are the fully normalized even zonal spherical harmonic coefficients of GRS80. n=2 gives

$$\bar{J}_2^{(N)} = -\frac{J_2^{(N)}}{\sqrt{5}} \tag{6}$$

Here (ϕ, λ) are the spherical latitude and longitude of the computation point, γ is the normal gravity at the telluroid.Telluroid is the surface whose height above a geocentric reference ellipsoid, normal height H^N, is the same as the height of the terrain above the quasigeoid. The quasigeoid is a non-equipotential surface of the Earth's gravity field that coincides reasonably closely with the geoid, up to about 3.4 m in the Himalayas (see e.g. Rapp 1997). H is the orthometric height of the computation point (P), h is the ellipsoidal height of point P, $\Delta g_{Bougner}$ is the Bouguer gravity anomaly which is an approximation to the correct value $\bar{g}-\bar{\gamma}$, $\bar{\gamma}$ is the average value of normal gravity between the point Q on the ellipsoid and the point Q' on the telluroid, both points corresponding to the computation point P, and $\bar{g}$ is the mean gravity averaged along the curved plumb line inside the topography. Figure 1 shows the geometric relations between ellipsoid, geoid and telluroid surfaces and the normal and orthometric heights. Note that the difference between orthometric and normal height is at the same time the difference between the height anomaly ζ and the geoid height N. a is the equipotential scale factor of EGM2008, GM is the gravity-mass constant of EGM2008, the coefficients $\bar{J}_{nm}$ and $\bar{K}_{nm}$ are fully normalized geopotential coefficients of degree n and order m of EGM2008 in the non-tidal system, $J_2^{(N)} = 0.108263 \times 10^{-2}$ for the GRS80 reference system and $\bar{P}_{nm}(\sin\varphi)$ are the normalized Legendre's polynomials.

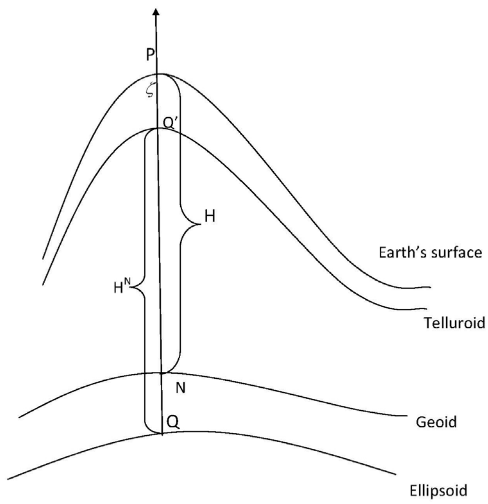

Figure 1. Geoid height (N) and height anomaly (ζ), orthometric height (H) and normal height (H^N)

Definition of the geoid is complicated by the permanent deformation of the Earth caused by the presence of the Sun and the Moon. Consideration of these permanent tidal effects has led to the definition of three types of geoids (see e.g. Ekman, 1989): i) Tide-free or non-tidal (geoid would exist for a tide-free Earth with all direct and indirect effects of the Sun and Moon removed), ii) Mean-tide (geoid would exist in the presence of the Sun and the Moon), and iii) Zero-tide (geoid would exist if the permanent direct effects of the Sun and Moon are removed, but the indirect effect component related to the elastic deformation of the Earth is retained). We note that the EGM2008 offers a non-tidal model (see the link in page 11). We performed our computations of EGM2008 using its non-tidal geopotential coefficient model since the GPS/levelling height anomalies used in this study were referred to the same system. It should be noted that formulas exist to convert the desired quantities (e.g. geoid heights) between different tidal systems (see e.g. Ekman, 1989).

The Bouguer anomaly in Eq. (1) can be computed from free-air anomalies when the station elevation is known (Heiskanen and Moritz, 1967, p. 131):

$$\Delta g_{\text{Bouguer}}(\phi,\lambda) = \Delta g_{\text{Free-air}}(\phi,\lambda) - 0.1119 H(\phi,\lambda) \tag{7}$$

where H is in meters and the gravity units are mGal. The Bouguer anomaly, in this study, is calculated as mean values on a grid whose spacing is compatible with the maximum degree of the EGM2008 (5' × 5'). Thus the calculation of the Bouguer anomaly is implied by the potential coefficient model and a Digital Terrain Model (DTM). The second term on the right hand side of Eq. (7) corresponds to the Bouguer plate. The major problem with Eq. (7) lies in the Prey reduction which is used in this study, i.e. the integration of gravity from the geoid to the Earth's surface. The Bouguer plate term is obtained simply as the gravity gradient produced by a straight plate that extends to infinity and having constant density of 2.67 g cm^{-3}. This means that the terrain is approximated by a flat plate of thickness H, the height of the point of interest above the sea level. Although better approximations exist (see e.g. Flury and Rummel, 2009) we chose this level of accuracy, for simplicity, for this study. There are approaches that usually rely on a modeled vertical gradient of gravity.

The free-air gravity anomalies in Eq. (7) can be computed from EGM2008 geopotential coefficients using (Heiskanen and Moritz, 1967, p. 108):

$$\Delta g_{\text{Free-air}}(r,\phi,\lambda)=\frac{GM}{r^2}\sum_{n=2}^{n_{\max}}(n-1)\left(\frac{a}{r}\right)^n\sum_{m=0}^{n}\left(\bar{C}_{nm}\cos m\lambda+\bar{S}_{nm}\sin m\lambda\right)\bar{P}_{nm}(\sin\phi) \tag{8}$$

The two remaining terms in Eq. (1) can be derived from Eq. (2) as (see Rapp, 1997; Nahavandchi, 2002):

$$\frac{\partial\zeta}{\partial r}h(r,\phi,\lambda)=-\frac{GM}{r^2\gamma}h\sum_{n=0}^{n_{\max}}(n+1)\left(\frac{a}{r}\right)^n\sum_{m=0}^{n}\left(\bar{C}_{nm}\cos m\lambda+\bar{S}_{nm}\sin m\lambda\right)\bar{P}_{nm}(\sin\phi) \tag{9}$$

and the second term becomes (see Rapp, 1997; Nahavandchi, 2002):

$$\frac{\partial\zeta}{\partial\gamma}\frac{\partial\gamma}{\partial h}h(r,\phi,\lambda)=0.3086h\left[\frac{GM}{r\gamma^2}\sum_{n=0}^{n_{\max}}\left(\frac{a}{r}\right)^n\sum_{m=0}^{n}\left(\bar{C}_{nm}\cos m\lambda+\bar{S}_{nm}\sin m\lambda\right)\right]\bar{P}_{nm}(\sin\phi) \tag{10}$$

Rapp (1997) rewrote Eq. (1) in the following form:

$$N(\varphi,\lambda)=\zeta_E(r,\varphi,\lambda)+C_1(r,\varphi,\lambda)+C_2(\varphi,\lambda) \tag{11}$$

where

$$C_1(r,\varphi,\lambda)=\frac{\partial\zeta}{\partial r}H+\frac{\partial\zeta}{\partial\gamma}\frac{\partial\gamma}{\partial H}H \tag{12}$$

and

$$C_2(\varphi,\lambda)=\frac{\Delta g_{\text{Bouguer}}}{\bar{\gamma}}H \tag{13}$$

where h in Eq. (12) has been replaced by H without significant error because the coefficients of H are small values and H-h is on the order of 10 m. To improve computation efficiency, C_1 and C_2 can be calculated globally on a grid and expanded into a spherical harmonic series. Spherical harmonics computations allow for an efficient evaluation once the spherical harmonic coefficients are determined. These correction terms can then be added to the value of the height anomaly ζ_E in Eq. (11) to obtain the geoid height.

It should be stated that Eq. (1) and subsequently Eqs. (11)–(13) involve an approximation ofthe mean gravity $\bar{g}$ averaged along the curved plumb line inside the topography.

Note that usually one puts $\Delta g_{\text{Bouguer}} = \bar{g} - \bar{\gamma}$. There are various levels of approximation in the determination of $\bar{g}$. The determination of $\bar{g}$ is based on knowledge and assumptions about the shape and density of topographic masses. Heiskanen and Moritz (1967, p. 327) based the well-known relation, between the geoid-quasigeoid separation and the Bouguer gravity anomaly, Eq. (1a), on Helmert's definitions. This level of approximation is used in this study; however, this relation can be far from the physical reality in very topographically rugged and mountainous areas. Flury and Rummel (2009) extended this well-known relation to a more rigorous approach accounting for the contribution of the attraction of topographic masses on the mean gravity $\bar{g}$ along the plumb line based on high-resolution DTM models. Their results showed smaller separation between height anomalies and the geoid heights in mountainous terrain, only 30 cm at the highest summit of 3798 m, while results based on approximations were often larger by several decimeters.

3. Numerical investigations

The test area for the geoidal height computation is limited by latitudes 30° N to 31° N and longitudes 54° E to 56° E. The area is located in Iran. The elevation in this area varies from 1290 to 4300 m. Figure 2 shows a map of the test area including GPS/levelling stations. The elevation of GPS/levelling stations varies from 1400 to about 2000 m. GPS/levelling data are used to examine the efficiency of the geoidal height computations through height anomaly. The GPS/levelling derived geoids are computed by the well-known formula

$$N = h - H \tag{14}$$

where h is the ellipsoidal height computed from GPS, and H is the orthometric height computed from precise levelling. There are 33 GPS/levelling stations in the study area. Although there is not enough information to assess the accuracy of the orthometric heights, Hamesh (1991) has estimated that the standard deviation of the orthometric heights in an absolute sense is around 70 cm. This is due to the sea surface topography (the height datum refers to multiple tide gauges, which are affected by the sea surface topography), presence of various systematic errors in observations and uncertainty about the definition and establishment of the height reference system used in adjustment of the network (Hamesh 1991). The accuracy of the GPS heights is estimated to be around 25 cm

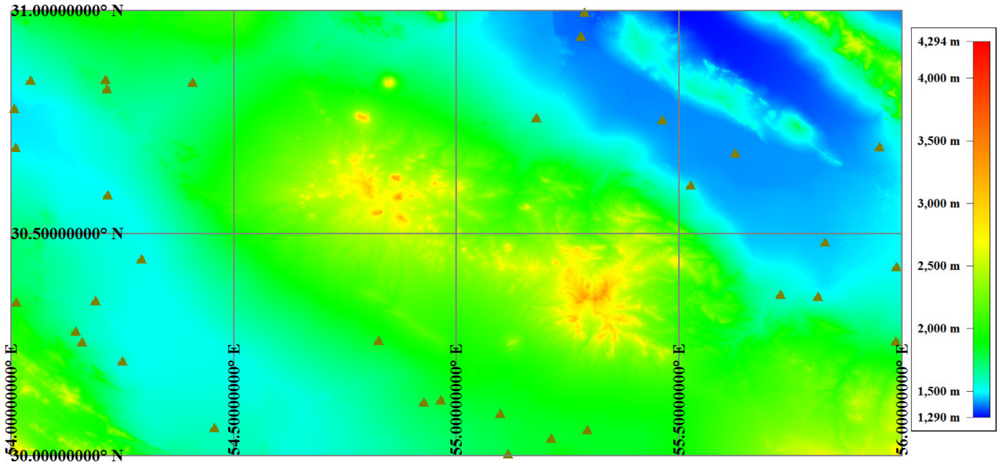

Figure 2. Distribution of the 33 GPS / levelling stations in the study area. The elevations are in m.

(Nilforoshan 1995). Note that standard deviations of the orthometric heights are undesirably high, but they are the only heights available for this project in the study region. This would mean that large differences between the EGM2008 and the GPS/levelling geoid heights could be expected.

Hamesh (1991) computed a gravimetric geoid model of Iran using a modification of Stokes' formula (remove-compute-restore method) combining the short wavelength contribution from terrestrial gravity (11000 grid observations with resolution of 110 × 160 arc-seconds) using Stokes' formula and height data (1 km × 1 km) with long wavelength contribution from the OSU89B global geopotential model (Rapp and Pavlis, 1990) to degree and order 360. The application of Stokes' formula requires that the disturbing potential is harmonic outside the geoid. This is satisfied by removing the effects of topographic and atmospheric masses or moving them inside the geoid. The effects of masses are then restored after applying Stokes' formula. The topographic and atmospheric effects, mentioned above were corrected in the Hamesh (1991) geoid model. The topographic and atmospheric correction terms were the basic equations of the remove-compute-restore technique formulated by Moritz (1980, Eqs. (48–32), (48–29) and (49–13)). The standard deviation and RMS of the differences between the gravimetric geoid model of Hamesh (1991) and the 33 points of GPS/levelling geoid used in this study are ±47 cm and ±70 cm.

We computed geoid heights through the height anomalies using Eq. (11). The DTM used in the computation is generated using the Geophysical Exploration Technology (GETECH) 5' 5' DTM (GETECH 1995). Geopotential coefficients are taken from the global EGM2008 (Pavlis et al., 2008) to the spectral degree of n_{max}=2160. The spatial resolution corresponding to degree 2160 is 180°/2160 = 1°/12 = 5 minutes (5' × 5'). To assess the accuracy of the EGM2008, the results of comparison of the EGM2008 and the GPS/levelling geoid heights globally at 12305 stations after the linear trend is removed was reported by Pavlis et al. (2008). The standard deviation of differences is ±10.3 cm (Pavlis et al., 2008). It should be noted that global error estimates for EGM2008 cannot be used directly to judge if this model is best for a certain region. Computations in this study are based on the software package developed by the second author. Parameter definitions are as follows: a = 6378.1363 km, GM = 3.986004415 × 10^{14} m^3s^{-2}, R = 6371.008771 km, and γ = 979.8 Gal.

Interested readers may refer to the EGM2008 web page which provides pre-computed geoid height values, a self-contained suite of coefficient files, and FORTRAN software. To compute geoid heights from a spherical harmonic synthesis of the EGM2008 non-tidal spherical harmonic coefficients and its associated correction model (height anomaly to geoid height correction term), one may use the FORTRAN harmonic synthesis program, hsynth_WGS84.f. The geoid heights can be computed at any latitude/longitude coordinate pair listed in a coordinate input file (such as INPUT.DAT).

(see http://earth-info.nga.mil/GandG/wgs84/gravitymod/egm2008/egm08_wgs84.html).

All synthesis software, coefficients, and pre-computed geoid grids provided in the link above assume a non-tidal system. The EGM2008 FORTRAN program was tested and compared with our software package, revealing only insignificant differences due to computer-dependent algebra.

The first step in determination of geoid heights is to calculate height anomalies using Eq. (2). We then compare the height anomalies with GPS/levelling derived geoid heights at 33 GPS/levelling stations. Standard deviation and RMS of the differences are ±50 cm and ±98 cm. Note that in this comparison we have not yet used any correction terms for the derived height anomalies.

The next step is calculation of the C_1 and C_2 terms. The DTM is given in a global grid. Evaluation of the Bouguer anomaly can also be done in a grid of mean values. The spacing of this grid is compatible with the maximum degree of EGM2008 (5' × 5'). The free-air anomalies $\Delta g_{\text{Free-air}}$ should be determined at the surface of the Earth; however, for simplicity they are first computed in a grid format on the surface of the ellipsoid. Then, the

C_1 and C_2 terms are computed on a grid with mean values in 5' × 5' cells. The GETECH DTM and the EGM2008 gravity model are used to estimate the C_1 and C_2 terms. The statistics of these two correction terms are given in Table 1. Absolute maximum values of 14 cm and 41 cm are found for the C_1 and C_2 correction terms. The result of the computations shows that both the C_1 and C_2 terms are significant and should always be applied. The C_1 term is around 21% of the magnitude of the C_2 term on average. The RMS of ±33 cm and SD of ±4 for the C_2 term (the Bouguer anomaly-elevation term) also indicate large values, on average, for the whole test area.

Table 1. The statistics of the C_1 and C_2 correction terms for 33 GPS / levelling stations in cm.

	C_1 correction term	C_2 correction terms
Min	–14	–41
Max	–2	–26
Mean	–7	–33
SD	±4	±4
RMS	±8	±33

Finally, the C_1 and C_2 terms are applied to the height anomalies, and using Eq. (11) the geoid heights are computed using the height anomalies. The results are then compared at the 33 GPS/levelling stations. Table 2 shows the statistics of differences between these two geoids.

Table 2. The statistics of differences between the GPS / levelling derived geoid heights and geoid heights computed using height anomalies at 33 GPS / levelling stations in cm. Both C_1 and C_2 correction terms have been applied.

Min	Max	Mean	SD	RMS
–5	85	40	±35	±53

For comparison, the statistics of differences between the Hamesh (1991) gravimetric geoid heights at the same 33 GPS/levelling stations are given in Table 3. Note that the Hamesh (1991) geoid was made by Stokes integration of terrestrial gravity anomalies. The gravity data were represented by 110 × 160 arc-seconds block values. These grid blocks are smaller than the 5' × 5' grids for the EGM2008. Geoid height commission error of OSU89B, used in Hamesh (1991) geoid model, is 53.6 cm at degree 360. In order to better quantify the results of the comparisons of Hamesh (1991) and the EGM2008, the RMS commission error of EGM2008 is 11.1 cm at degree 2160. The commission error is the error of the model. The limited spectral content of the model causes the omission error. It is included in the reported commission errors for the EGM2008 and OSU89B.

Table 3. The statistics of differences between the GPS / levelling derived geoid heights and the Hamesh (1991) gravimetric geoid model at 33 GPS / levelling stations in cm.

Min	Max	Mean	SD	RMS
–12	124	52	±47	±70

Considering the values given in Table 2 and the fact that the standard deviation of the differences before applying C_1 and C_2 correction terms is ±50 cm, we learn that the approach used in this study and the two correction terms yields geoid heights that fit the GPS/levelling data better than if no correction terms are applied, as was expected. The values given in Table 2 and 3 also show very good results at GPS/levelling stations compared with the results of gravimetric geoid heights using terrestrial gravity anomalies referred to the same stations. Note that the Hamesh (1991) gravimetric geoid is an old model which was available for this study. The newer gravimetric geoid models probably provide better results.

It should be noted here that the zero term (ζ_0) due to the difference between the gravity-mass constant GM of the geopotential model of the Earth and the gravity-mass constant GM_0 of the normal field is also included in the computations of the geoidal heights through height anomalies. ζ_0 is computed according to the well-known formula (Heiskanen and Moritz,1967, p. 101):

$$\zeta_0 = \frac{GM - GM_0}{R\gamma} - \frac{W_0 - U_0}{\gamma} \quad (15)$$

where GM = 3.986004415 × 10^{14} m^3s^{-2} and GM_0 and U_0 correspond to the normal gravity field on the surface of the normal ellipsoid. For the GRS80 ellipsoid we have GM_0 = 3.986005000 × 10^{14} m^3s^{-2} and U_0 = 62636860.85 m^2s^{-2}. The constant gravity potential for the geoid is W_0 = 62636856.00 m^2s^{-2}. Based on the above conventional choices, the zero degree term from Eq. (15) yields the value $\zeta_0 = -0.442$ m. A zero degree term of –41 cm is published at the EGM2008 web page referenced to WGS84 with slightly different parameters than those used in this study.

4. Discussion

An approach to determining geoid heights from a global geopotential coefficient model alone, here EGM2008, is investigated. In this approach, the height anomalies ζ_E are calculated first. The height anomalies are subsequently converted to geoidal heights using two correction terms, one of which representing the height anomaly gradient terms and the second one the height anomaly-geoidal height difference. This approach is very suitable and simple to use from the computational point of view compared with the classical gravimetric Stokes' approach of geoid height determination using terrestrial gravity data, which requires a lot of computational efforts.

The investigated approach for the geoidal height determination is examined at 33 GPS/levelling stations. Good agreement with the GPS/levelling data was demonstrated, although we know that some short-wavelength information is missing compared to the well-known Stokes integration. Surprisingly, the standard deviation of differences between geoid heights determined through height anomalies (including the two correction terms) and the GPS/levelling geoid is ±35 cm, while it was ±47 cm with the gravimetric geoid heights computed through Stokes integration (Hamesh model) compared at the same stations. Note that the Hamesh (1991) model was based on an older geopotential model and might be improved. These computations should also be done in other test areas.

Computation of regional gravimetric geoid models with reasonable accuracy, in countries where terrestrial gravity data are sparse and they have been observed over a long time using different equipment, methods and reference frames, is a difficult task requiring special care. Nowadays, with increasing accuracy of the geopotential coefficients derived from global Earth gravity models and increasing maximum degree of expansion to a higher degree (as in the case of this study using EGM2008 to degree 2160), an alternative is to compute geoid heights by combining limited ground gravity data with the EGM2008 global geopotential model and a digital terrain model.

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